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Osamu Majima & Hiroshi Suemasu

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An interface element with continuous traction to analyze delamination propagation

OSAMU MAJIMA * and HIROSHI SUEMASU

Department of Mechanical Engineering, Faculty of Science and Technology, Sophia University, 7-1, Kioicho Chiyodaku, Tokyo, 102-8554, Japan

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Abstract—An interface element which accounts for the properties of both material strength and fracture mechanics is proposed to numerically simulate the damage propagation process in composite laminates. Several element refinements, which ensure the smooth convergence of the solution during the calculation of damage, are incorporated through a user subroutine in the commercially available finite element package program (MARC). The element was successfully applied to the fundamental problems of stable crack propagation and crack initiation. The propagation process of multiple interlaminar delaminations in circular axisymmetric nonlinear plates subjected to a quasi-static transverse load has been solved to demonstrate the applicability and efficiency of the present element in a complex delamination propagation problem. Because a continuous stress distribution with respect to surface separation along the crack path is assumed on the interface, smooth and stable convergence of the solution of crack propagation can be realized with a fairly coarse mesh compared to the mesh of the model when a discrete interface traction force is assumed only at the nodes on the interface. Since the interface element considers the energy dissipated during the formation of the crack surface, it may be applied to the simulation of a dynamic failure process.

Keywords: Composite laminate; delamination; interface element; finite element method; simulation of damage propagation.

1. INTRODUCTION

Structures made of composite laminates, particularly those made of carbon fiber reinforced plastics (CFRP), have been used extensively in many industrial applications, such as aircraft and space structures. It is known that structures made of composite laminates are susceptible to impact damage including delamination and transverse cracking. It is difficult to detect this impact damage from the outside of the structure. Furthermore, such damage induces severe reduction of compressive

*To whom correspondence should be addressed. E-mail: o-majima@me.sphoia.ac.jp

strength. Therefore, the mechanism of damage propagation due to impact loading must be well understood, and then, it is important to develop a method to accurately simulate this damage propagation.

There have been previous attempts using numerical analysis to resolve the damage process based on the concept of material strength [1, 2]. However, it is necessary to consider fracture mechanics in the case of propagation of crack-type damage, as in interlaminar delamination. The authors of this paper believe it is suitable to use a cohesive model because a crack propagates along only the interlaminar surface in the case of interlaminar delamination. That is, we consider the relationship of traction to relative displacement along the delamination surface, where the traction is zero when fracture occurs.

To determine the properties of these cohesive models, especially the relationship between traction and relative displacement, various modeling methods have been proposed which satisfy the fracture mechanics condition and consider softening effects such as the failure of the fiber and bridging. Cui and Wisnom [3], Allix *et al.* [4], and Yan *et al.* [5] assumed elasto-plastic or elasto-perfectly plastic behavior approximating the traction in the plastic zone using an appropriate function. Mi *et al.* [6], Wisheart and Richardson [7], de Moura *et al.* [8], Sprenger *et al.* [9], Alfano and Crisfield [10], Borg *et al.* [11], and Crisfield and Alfano [12] considered the softening effect using the bi-linear relationship of traction and relative displacement. Chen *et al.* [13] proposed a cohesive model where traction was approximated using a cubic polynomial function. Chaboche *et al.* [14] and Sayed and Sridharan [15] applied a model used to simulate crack propagation in metal.

The softening behavior is extremely complicated; therefore, the validity of the above researchers' modeling has been confirmed by comparing their results with experimental results [16, 17]. On the other hand, it has been indicated that the behavior of delamination propagation is little influenced by the modeling if the fracture mechanics condition is satisfied [15, 18, 19].

When we attempt to treat the impact damage of composite laminates as a problem of dynamic response, it is necessary to use modeling that considers the energy dissipated during formation of the crack surface. The authors previously proposed a method that considers such energy by introducing a fracture element using a nonlinear spring, which satisfies the material strength and fracture mechanics conditions in the finite element analysis [18, 19]. But, it was necessary to use a very fine finite element mesh to obtain a satisfactory result. Therefore, we wanted to develop a more efficient method for solving impact problems, which require huge numerical effort. Thus, we proposed a more appropriate method for static problems of crack propagation with the consideration that a spring at the crack tip is too stiff [20]. Unfortunately, this method could not properly estimate the energy dissipated during formation of the crack surface.

In this study, an interface element which accounts for the properties of material strength and fracture mechanics is proposed to numerically simulate the multiple delamination propagation process of composite laminates. This delamination propagation process occurs in the impact damage process. Continuously distributed stresses along the process zone can be simulated using an interface element. This method is applicable to dynamic problems because it can properly estimate the energy dissipated during formation of the crack surface. To confirm that this method is valid, the interface element is applied to a problem for which an exact solution has already been given by linear fracture mechanics. It is shown that this method can approximate crack initiation by applying to the problems accompanying the initiation of delamination of composite laminates. The propagation process of multiple interlaminar delaminations in a circular axisymmetric laminate subjected to a transverse concentrated load has been analyzed to demonstrate that this result is more likely to converge than the results of previous models [18–20], and that the analysis of the delamination propagation can be done smoothly.

2. MODELING OF INITIATION AND PROPAGATION OF A CRACK USING AN INTERFACE ELEMENT

It is necessary to satisfy the condition of material strength to simulate the initiation of a crack at the interlaminar of a composite laminate. It is also necessary that the fracture mechanical property is properly incorporated in the analysis to simulate the propagation process of delamination. Furthermore, an analysis technique that can consider the energy dissipated during the formation of the crack surface is necessary to handle the damage process of a composite laminate as a dynamic problem.

An interface element of volume zero before deformation, shown in Fig. 1, is defined along the surface where the crack is expected to propagate. Figure 1 shows a deformed element shape: that is, double nodes l and l', m and m', n and n' have the same coordinates, respectively, before deformation. The relative displacements along the normal and tangential directions of the crack surface in the element are defined as

$$\Delta u_n = \sum_i \phi_i(\xi)\Delta u_{ni} \Delta u_s = \sum_i \phi_i(\xi)\Delta u_{si}, \tag{1}$$

where $\Delta u_{ni} = u_{ni'} - u_{ni}$ and $\Delta u_{si} = u_{si'} - u_{si}$ are the relative displacements of the double nodes, $\varphi_i(\xi)$ is the shape function which is identical to that along the boundary of the finite element adjacent to the interface element, and $\xi(-1 \leqslant \xi \leqslant 1)$ is a local coordinate defined in the element.

Deformation of the crack tip area in which the deformation is large, that is, the process zone, is simulated using the interface element. In the problem of delamination process of composite laminates, the area of the process zone is relatively small compared to that of crack surface. Then, we consider unloading as an overall fracture process by the formation of the crack surface [12]. Therefore, we assume that the tractions applying to the surface along which the crack propagates are defined as an explicit function of the relative displacement. Considering that the

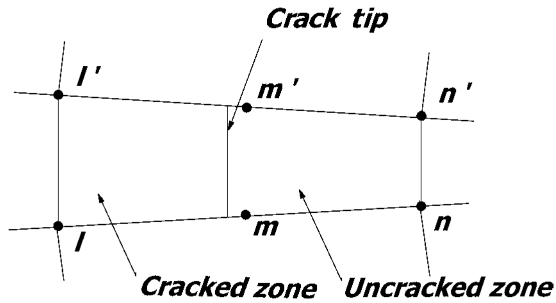

Figure 1. Interface element.

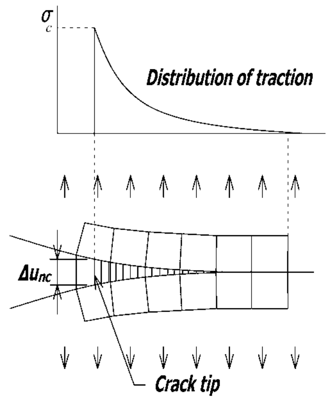

Figure 2. Distributed traction along the crack tip interface.

overall behavior of delamination propagation is mainly controlled by the properties of material strength and fracture mechanics [15], we assume that the maximum traction at the crack tip is equal to the material strength when the delamination propagates, and that the work done at the crack surface until failure is equal to the fracture toughness of the material. When the deformation in the element approaches the critical value satisfying these conditions, the crack propagates to that position and the traction in the area crack propagated is zero (see Fig. 2). It is ideal that the interface element does not deform during elastic deformation because we assume that elastic deformation is represented by a normal finite element, and the interface element simulates the locally large deformation at the crack tip area, the process zone. But too large an initial slope of the relationship of traction and relative displacement in the interface element creates problems in the numerical calculations.

Considering the above discussion, we assume that the tractions along the normal and tangential directions in the element are represented as

$$\sigma(\Delta u_n) = \begin{cases} k_n\left[(\Delta u_n + \Delta u_{n0})^\alpha - \Delta u_{n0}^\alpha\right] & (0 \leqslant \Delta u_n < \Delta u_{nc}) \\ k_{n0}\Delta u_n & (\Delta u_n < 0) \end{cases}$$

$$\tau(\Delta u_s) = \pm k_s\left[(|\Delta u_s| + \Delta u_{s0})^\beta - \Delta u_{s0}^\beta\right] \quad (|\Delta u_s| < \Delta u_{sc}), \tag{2}$$

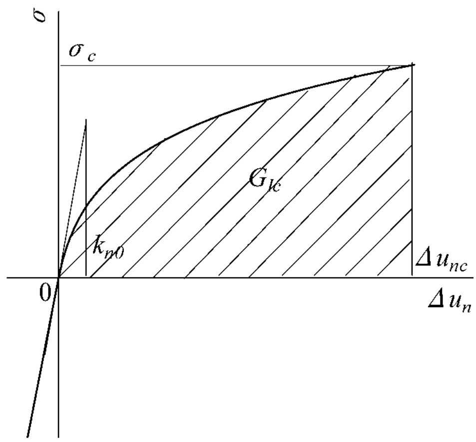

Figure 3. Typical relationship between traction and relative displacement of interface element.

α and β ($0 < \alpha < 1, 0 < \beta < 1$) are arbitrary constants, Δu_{n0} and Δu_{s0} are small constants introduced in order that the slope of tractions at $\Delta u_n = 0$ or $\Delta u_s = 0$ have finite values, and Δu_{nc} and Δu_{sc} are the critical relative displacements in which the crack propagates in mode I or mode II. The relationship between traction and relative displacement in the normal direction is shown in Fig. 3. Therefore,

$$\sigma(\Delta u_{nc}) = \sigma_c \int_0^{\Delta u_{nc}} \sigma(x)\,\mathrm{d}x = G_{Ic}\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta u_n}\right]_{\Delta u_n=0} = k_{n0}, \tag{3}$$

where σ_c and G_{Ic} are, respectively, the interlaminar tensile strength and fracture toughness of mode I, k_{n0} and α are constants determined for convergence of the numerical calculation and the condition that can be sufficiently admissible for elastic deformation in engineering applications. The initial slope k_{n0} must be given large values so that deformation of the interface element is sufficiently small in elastic deformation. A small α can simulate the behavior for small work hardening. Equation 3 reduces to

$$\begin{aligned} \Delta u_{n0} &= \left(\frac{k_{n0}}{\alpha k_n}\right)^{1/(\alpha-1)}, \\ \Delta u_{nc} &= \left[\frac{\sigma_c}{k_n} + \left(\frac{k_{n0}}{\alpha k_n}\right)^{\alpha/(\alpha-1)}\right]^{1/\alpha} - \left(\frac{k_{n0}}{\alpha k_n}\right)^{1/(\alpha-1)}, \\ &\frac{1}{\alpha+1}\left\{\left[\frac{\sigma_c}{k_n} + \left(\frac{k_{n0}}{\alpha k_n}\right)^{\alpha/(\alpha-1)}\right]^{(\alpha+1)/\alpha} + \alpha\left(\frac{k_{n0}}{\alpha k_n}\right)^{(\alpha+1)/(\alpha-1)}\right\} \\ &- \left(\frac{k_{n0}}{\alpha k_n}\right)^{\alpha/(\alpha-1)}\left[\frac{\sigma_c}{k_n} + \left(\frac{k_{n0}}{\alpha k_n}\right)^{\alpha/(\alpha-1)}\right]^{1/\alpha} - \frac{G_{Ic}}{k_n} = 0. \end{aligned} \tag{4}$$

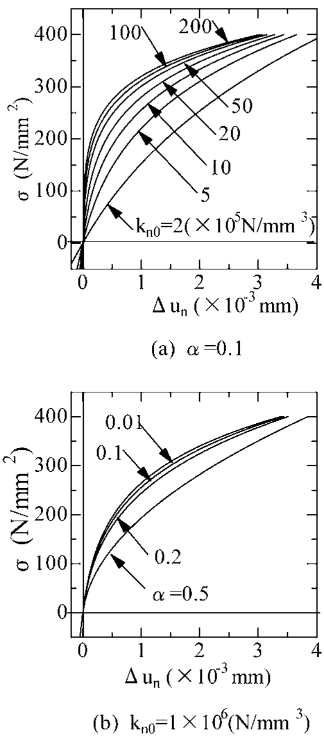

Figure 4. Influences of k_{n0} and α on the relationship between traction and relative displacement. ($\sigma_c = 400$ MPa, $G_{Ic} = 1.0$ kJ/m^2).

The parameters k_{n0}, Δu_{n0} and Δu_{nc} can be determined by solving these nonlinear simultaneous equations. As an example, the relationships between σ and Δu_n for various values of k_{n0} and α are shown in Fig. 4 for $\sigma_c = 400$ MPa and $G_{Ic} = 1.0$ kJ/m^2. The property of traction along the tangential direction can also be determined by a similar procedure.

When Δu_n or Δu_s reaches the critical relative displacement Δu_{nc} or Δu_{sc}, the crack is considered to propagate to that position. The position ξ_c of the crack tip in the element in which the crack tip is contained is

$$\Delta u_n(\xi_c) = \Delta u_{nc} \text{ or } \Delta u_s(\xi_c) = \Delta u_{sc}. \tag{5}$$

For the contact problem, only the following compressive reaction is applied along the crack surface in the area in which the crack has propagated

$$\sigma(\Delta u_n) = \begin{cases} 0 & (\Delta u_n \geqslant 0) \\ k_{n0}\Delta u_n & (\Delta u_n < 0) \end{cases} \qquad \tau(\Delta u_s) = 0, \tag{6}$$

k_{n0} is the parameter that provides the contact property after the crack propagates.

When equation (5) is satisfied in the area that the crack has not propagated, the crack propagates and tractions vary from equation (2) to equation (6). Therefore, the deformation energy stored in that position before propagation dissipates by the formation of the crack surface.

The work done by the tractions applied to the interface element is

$$V = \int_A \left[\int_0^{\Delta u_n} \sigma(x)\,\mathrm{d}x + \int_0^{\Delta u_s} \tau(x)\,\mathrm{d}x \right] \mathrm{d}A. \qquad (7)$$

If we apply the normal finite element formulation to this equation, we get the tangential stiffness equation of the interface element.

The above interface element is incorporated into commercial FEM code (MARC) as a user-defined element.

3. PROPAGATION OF A CIRCULAR CRACK IN AN INFINITE ISOTROPIC MEDIUM

Propagation of a circular crack in an infinite isotropic medium has been analyzed to confirm the validity of the modeling of crack propagation by the present method using the interface element. The material properties of the isotropic medium are $E = 100$ GPa, $\nu = 0.3$, $\sigma_c = 400$ MPa and $G_{Ic} = 1.0$ kJ/m^2. A uniform load is applied to a part of the inner surface of the circular crack, as shown in Fig. 5. The crack stably propagates in mode I in this problem. The radius of the initial crack is 2 mm. The load is applied to the center part, which has a radius of 1 mm. This problem is an axisymmetric problem; therefore, an 8-node axisymmetric solid element is used. Because of the symmetry, only one-half of the area is analyzed. Interface elements are added in the circular area whose radius is 20 mm. An example of the FEM model is shown in Fig. 6. The numerical calculations are done for $\alpha = \beta = 0.1$, $k_{n0} = k_{s0} = 1.0 \times 10^6$ N/mm^3.

Figure 7 shows the convergence of the result according to the size of the interface element. The exact solution using linear fracture mechanics (labeled 'Theory' in

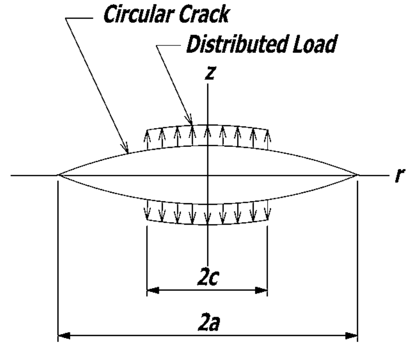

Figure 5. A circular crack in an infinite isotropic medium.

the figure) is shown for comparison. The crack radius is smaller than the exact solution because this analysis considers the process zone in the crack tip area. The results approximately converge when the element size is 1.0 mm. But the crack propagation is not totally smooth because the propagation behavior is influenced by the position of the crack tip in the interface element. It is considered that the crack propagation is almost smooth for the smaller element sizes of 0.25 mm.

Figure 8 shows the change of the distribution of the normal displacement in the crack tip area when the load is incrementally increased for element sizes 1.0 mm

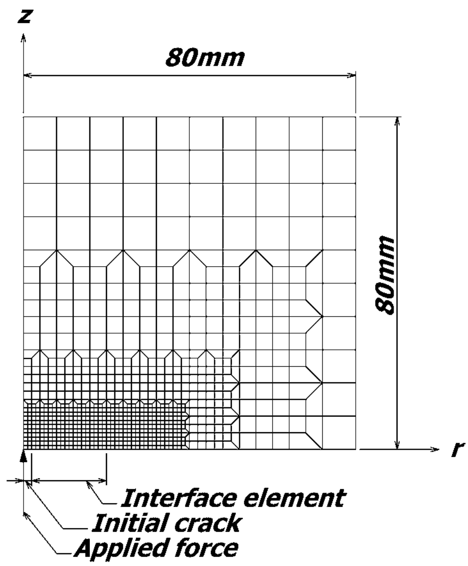

Figure 6. A finite element model of a homogeneous axisymmetric body with a circular crack.

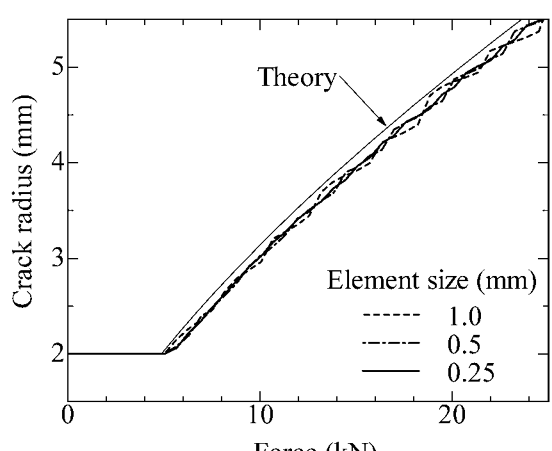

Figure 7. Relationships between applied force and crack radius of a circular crack in an infinite medium.

and 0.25 mm. The vertical dashed lines in the figure show the element boundary. The locally varying deformation in the crack tip area is represented by one or two elements when the element size is 1.0 mm. Since the deformation is approximated by a quadratic function in each element when an 8-node isoparametric element is used, the exact deformation cannot be properly represented when the finite element mesh is not sufficiently fine; therefore, the distribution of the deformation is not smooth at the element boundary. That is, the deformation of the process zone can easily be represented by two elements, but the figure shows that one element cannot approximate the deformation. Therefore, it is shown that the displacement slightly decreases when the crack tip propagates across the element, and so the crack tip does not smoothly move across the element. The deformation in the crack tip area can be nearly smoothly represented when the element size is 0.25 mm because the process zone expands to two or more elements. But the propagation of the crack is slightly influenced by the position of the crack tip in the element in this case as well. It is necessary to use an infinite number of elements to obtain completely smooth crack propagation. The result shown here is thought to be sufficient for engineering use. In the analysis using the present method, it is necessary to determine the element

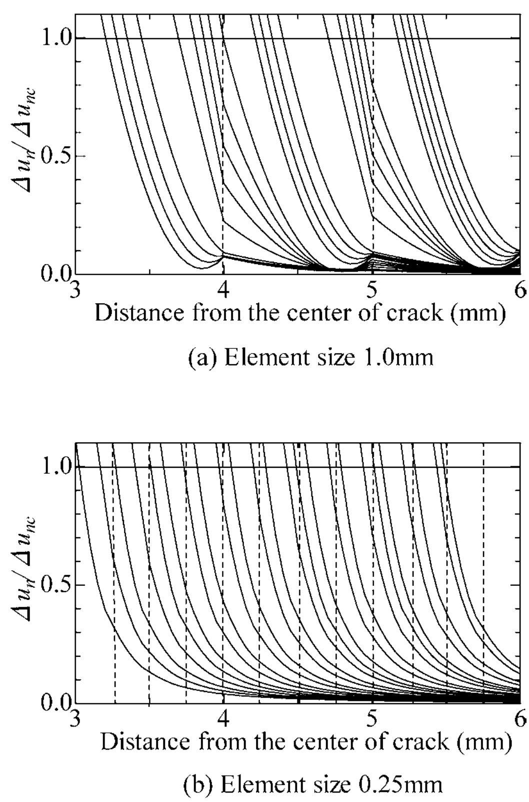

Figure 8. Distributions of relative displacement at the crack tip in an infinite medium.

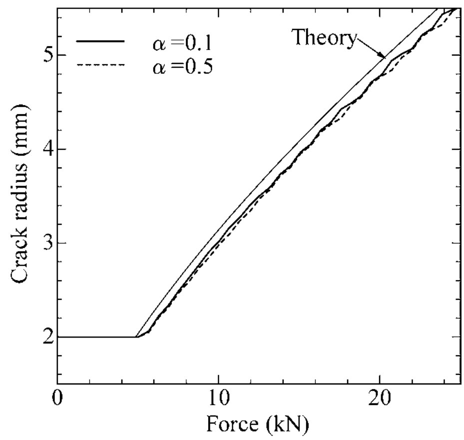

Figure 9. Effect of α on the relationship between applied force and crack radius in an infinite medium ($k_{n0} = 1.0 \times 10^6$ N/mm^2).

size so as to represent the extremely varying deformation at the crack tip area using more than two elements.

Figure 9 shows the results when $\alpha = 0.1$ and $\alpha = 0.5$, the element size at the crack tip area is 0.25 mm, and $k_{n0} = k_{s0} = 1.0 \times 10^6$ N/mm^3. It is recognized that the overall result is not influenced much by α. We also studied the effect of the value k_{n0}. The results, which are not shown here owing to limitations of space, are not influenced much by k_{n0}. That is, if the crack propagation is considered on the basis of fracture mechanics, the behavior is controlled by the energy stored until failure, and the effects of other parameters on the overall behavior are small.

4. APPLICATION TO PROBLEMS CONTAINING THE INITIATION AND PROPAGATION OF THE CRACK

The method presented is somewhat applicable to the problem containing not only crack propagation but also crack initiation. Consider the problem that a uniform tensile displacement is applied at the upper and lower surfaces of the tip of a laminated cantilever plate, and delamination initiates and propagates. The length and thickness of the plate are 200 mm and 4 mm, respectively. The displacement is applied to an area 0.5 mm from the tip of the cantilever plate. When the delamination is assumed to uniformly initiate and propagate along the center surface, this problem is analyzed as a plane strain problem. Considering the symmetry, only the upper half part of the plate is analyzed. A FEM model is shown in Fig. 10. The numerical calculations are done for $\alpha = \beta = 0.1$ and $k_{n0} = k_{s0} = 2.0 \times 10^6$ N/mm^3. This is considered a quasi-isotropic laminate made of CFRP, so it is analyzed as an in-plane isotropic and out-of-plane orthotropic homogeneous material. The material properties used in the analysis are shown in Table 1.

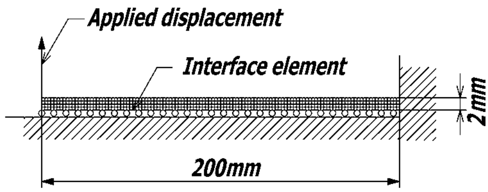

Figure 10. A finite element model of cantilever composite plate.

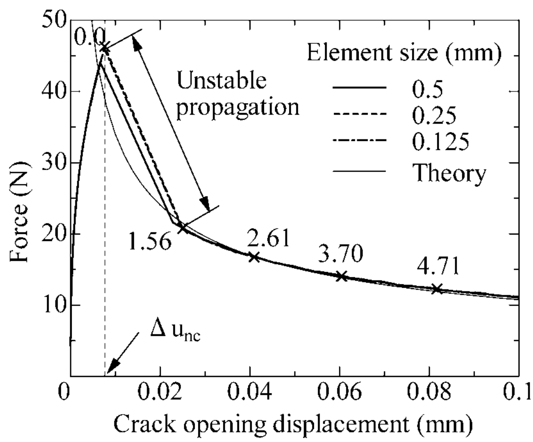

Figure 11. Relationships between applied force and crack opening displacement in cantilever composite plate.

Table 1.
Material properties of transversely isotropic plate

$E_x = E_y$	56.5 GPa
ν_{xy}	0.316
E_z	9.15 GPa
$\nu_{xz} = \nu_{yz}$	0.262
$G_{xz} = G_{yz}$	4.18 GPa
σ_c	32.6 MPa
τ_c	85.0 MPa
G_{Ic}	200 J/m^2
G_{IIc}	400 J/m^2

Figure 11 shows the relationship between the crack opening displacement and the force. The result of the analysis of the double cantilever beam (DCB) test using beam theory (labeled 'Theory' in the figure) is shown for comparison. Numbers in the figure show the crack length in each state when the element size is 0.25 mm. To represent the deformation of the process zone by the interface elements, the relationship between the crack opening displacement and the force is nonlinear before the crack opening displacement reaches the critical relative displacement. When the relative displacement reaches the critical value, the crack

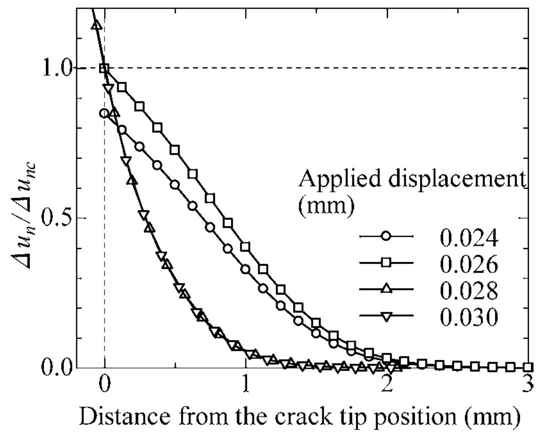

Figure 12. Distributions of relative displacement at the crack tip in cantilever composite plate.

initiates and unstably propagates for about 1.5 mm, greatly decreasing the force. The phenomenon of unstable propagation does not exist in the DCB test when a finite length initial crack exists. The relationship between the crack opening displacement and the force agrees well with the result of analysis of the DCB test using beam theory after the unstable propagation of the crack has been completed. Figure 11 shows that the result, including before and after unstable propagation, converges when the element size is 0.25 mm.

Figure 12 shows the distributions of the relative displacement in the crack tip area before and after unstable propagation for the element size 0.25 mm. The relative displacement has been normalized with the critical relative displacement. The applied displacement of 0.026 mm is the state just before the crack initiates. The stress distribution is controlled by the material strength property before the crack initiates, and the deformation of the interface elements is distributed over a wide area. The crack propagates unstably by 1.5 mm as soon as it initiates. The deformation of the interface element is restricted to the extremely local area near the crack tip (process zone) after the crack initiates.

5. PROPAGATION OF MULTIPLE INTERLAMINAR DELAMINATIONS IN AN AXISYMMETRIC LAMINATE SUBJECTED TO A CONCENTRATED LOAD

The propagation process of multiple interlaminar delaminations in a circular axisymmetric laminate is analyzed as an example of the propagation of interlaminar delamination in a laminate. The radius and thickness of the circular laminate are 50 mm and 4 mm, respectively. There are seven initial delaminations of radius 2 mm along the plate thickness in the center area. A uniform displacement is applied to the surface of the center at a radius of 1 mm. The material properties of the laminate are the same as those shown in the previous section.

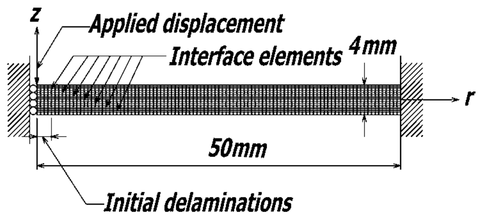

Figure 13. A finite element model of axisymmetric composite laminate with circular delaminations.

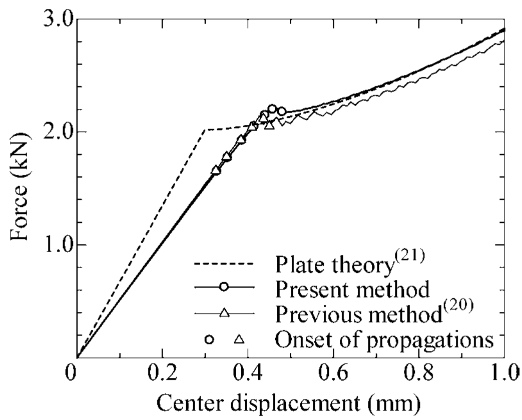

Figure 14. Relationships between center displacement and applied force of axisymmetric laminate with seven delaminations.

An example of the FEM model is shown in Fig. 13. The element size near the crack tip area is 0.25 mm. The delaminations are assumed to uniformly propagate along the planes which contain the initial delamination and are parallel to the center plane. Therefore, after adding interface elements along these whole planes, the model is analyzed as an axisymmetric problem. Initial delaminations are modeled using initially damaged interface elements. The numerical calculations are done as an axisymmetric but geometrical nonlinear problem where $\alpha = \beta = 0.1$, $k_{n0} = 2.0 \times 10^5$ N/mm^3. Eight-node axisymmetric solid elements are used.

The result is compared to the result of the previous method, which was modeled using a nonlinear concentrated spring [20]. Figure 14 shows the relationship between the center displacement at the surface opposite to the forced surface and the applied force. The result of the approximate analysis using the theory of plate bending [21] is shown for comparison. The forces at which the delaminations begin to propagate are shown on the force displacement curve.

Since the present method can continuously determine the position of the crack tip, the phenomenon that the propagation of crack depends on element size has been improved compared to the previous method, and so the result that the crack smoothly propagates in the same element size has been shown. This result seems to be an effect of continuous stress distribution. The delaminations have begun to propagate in order from the forced surface. There is a sudden decrease of force caused by unstable propagation of cracks when the delamination farthest from the forced surface becomes unstable. The behavior after all delaminations have begun to propagate agrees well with the result of the approximate analysis using the theory of plate bending for the range of force presented. The difference between the approximate analysis before the beginning of propagation of the delaminations is caused by the effect of shear deformation and the reduction of stiffness by the initial delamination.

As a result it has been confirmed that the behavior of propagation of multiple delaminations can be simulated using the model presented here.

6. CONCLUSION

In this study, an analytical technique that satisfies the properties of material strength and fracture mechanics has been proposed to analyze the propagation of interlaminar delaminations of a composite laminate. This method, which uses an interface element, is also applicable to the problem of impact damage with consideration of the energy dissipated during formation of the crack surface.

It has been confirmed that the present method is appropriate for application to the problem for which an exact solution has been given using the theory of linear fracture mechanics. It has also been confirmed that the present method can handle the problem of crack initiation, in which it is necessary to consider the material strength to some extent. This method can represent the continuous distribution of stress occurring on the crack surface. It is necessary to determine the element size so as to represent the deformation at the process zone with several interface elements because deformation is dominated by the degree of freedom of the shape functions.

The propagation of multiple interlaminar delaminations of an axisymmetric circular laminate subjected to a concentrated load was analyzed. The result was compared to a previous method using a concentrated spring element. The position of the crack tip can be continuously determined regardless of the mesh size of the present method, assuming a continuous stress distribution. By smoothly decreasing the stiffness between double nodes accompanying crack propagation, the results stably converge and a smooth equilibrium curve can be obtained.

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